MATH 347 HW 4

due October 2, at the beginning of class

Homework Guildlines

Obviously, your solutions need to be complete and correct, but to receive full credit your write-up should also satisfy the following:

- All the important logical steps in the proof should be present and fully explained.
- All assumptions should be clearly identified.
- Your solutions should be clear and concise. If a sentence does not further the reader's understanding of the solution then it has no place in your write up.
- Use full and grammatically correct English sentences. Mathematical symbols should be used only to render complex mathematical relationships into a readable form.

Moreover, in order to obtain full credit for the homework, you must write down, in the very least, an attempt at a solution for each problem.

Problems

Do the following problems from your book: 3.9, 3.26, 3.43, 3.48, 3.64. Also answer the following:

(1) Recall that the Fibonacci sequence is defined recursively by $F_1 = F_2 = 1$ and $F_n = F_{n-2} + F_{n-1}$. Show that F_{3n} is always even. What can you say about F_{4n}?

(2) Consider a circle with $2n$ dots placed on the circle. Suppose that n of these dots are red and the other are blue. Going around the circle clockwise, you keep track of how many red and how many blue dots you have passed. You consider it a good trip if at any moment along the trip the number of blue dots you have passed is at least the number of red dots. Show that no matter the configuration of the dots, you can always choose a starting point so that the trip will be successful.

(3) Your book claimed without proof (see pg 58) that, in a n-by-n checkerboard, the number of squares of with sides of length

$n+k-1$ is k^2. Prove this, thus showing the sum $\sum_{k=1}^{n} k^2$ is the total number of squares in a n-by-n checkerboard.